MATH 407 QUIZ	Problem 1	Name (Please print):
July 24, 2008	40 points	____________________

Problem:

$$\begin{array}{rlll} \text{maximize} & 3x_1 + x_2 & \text{w.r.t} & x \in \mathbf{R}^2_+ \\ \text{subject to} & x_1 - x_2 & \leq -1 & \\ & -x_1 - x_2 & \leq -3 & \\ & 2x_1 + x_2 & \leq 4 & \end{array}$$

Fill in the missing characters denoted by _ in the message below so that e-mailing the resulting message to Neos would solve the problem above. Do not fill in any text for characters that should be a space.

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Problem 3.9a in Vasek Chvatal's book: Linear Programming
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__  r2
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    x1        r2        -1             r3        2
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Solution:

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COLUMNS
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*
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    x2        r2        -1             r3        1
RHS
    b         r1        -1
    b         r2        -3
    b         r3        4
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MATH 407 QUIZ	Problem 2	Name (Please print):
July 24, 2008	40 points	________________

Problem: We consider a game where players X and Y simultaneously (and repeatedly) choose one of the following options: Rock, Paper, or Scissor. The following table gives the payoff from player Y to player X for each of the possible cases

		X		
		Rock	Paper	Scissor
	Rock	0	1	-1
Y	Paper	-1	0	1
	Scissor	1	-1	0

Write down a linear programming problem that corresponds to X choosing the optimal strategy under the assumption that Y discovers the strategy. Show that $\hat{y} = \hat{x} = (1/3, 1/3, 1/3)^{\mathbf{T}}$ is the optimal strategy for both X and Y.

Solution: The linear programming problem that corresponds to player X is

$$\begin{array}{rrl} \text{maximize} & z & \text{w.r.t} \quad z \in \mathbf{R}\ ,\ x \in \mathbf{R}^3_+ \\ \text{subject to} & x_1 + x_2 + x_3 & = 1 \\ & z - x_2 + x_3 & \leq 0 \\ & z + x_1 - x_3 & \leq 0 \\ & z - x_1 + x_2 & \leq 0 \end{array}$$

It suffices to show that the values of the dual and primal objectives are equal when $\hat{y} = \hat{x} = (1/3, 1/3, 1/3)^{\mathbf{T}}$. The primal objective value is equal to

$$\inf_{y \in \mathbf{P}(3)} (y_1, y_2, y_3) \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} = \inf_{y \in \mathbf{P}(3)} (y_1, y_2, y_3) \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0$$

Hence the primal objective value is zero at $\hat{x}$. The dual objective vlaue is equal to

$$\sup_{x \in \mathbf{P}(3)} (1/3, 1/3, 1/3) \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \sup_{x \in \mathbf{P}(3)} (0, 0, 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

Hence the dual objective value is zero at $\hat{y}$. Since $(1/3, 1/3, 1/3)^{\mathbf{T}} \in \mathbf{P}(3)$ (it is feasible) we conclude that it solves both the primal and dual problems.