

**Problem:** We are given the problem

$$\begin{aligned} & \text{maximize} && -5x_1 + 6x_2 - 9x_3 - 8x_4 && \text{w.r.t } x \in \mathbf{R}_+^4 \\ & \text{subject to} && x_1 + 2x_2 + 3x_3 + x_4 && \leq 5 \\ & && x_1 + x_2 + 2x_3 + 3x_4 && \leq 3 \end{aligned}$$

Fill in the missing characters below (denoted by underbars) so that the Octave/Matlab program performs one feasible pivot for the problem above:

```
A = [ ... % initial tableau
      1 , 2 , 3 , __ , __ , __ , 5
      1 , 1 , 2 , __ , __ , __ , 3
      __ , __ , __ , __ , 0 , 0 , 0 ...
];
format = '%8.3g';
fprintf('%8s', 'x1', 'x2', 'x3', 'x4', 's1', 's2', 'b', '\n');
r = __; c = __;
B = pivot(A, r, c, format);
```

**Solution:**

```
A = [ ... % initial tableau
      1 , 2 , 3 , 1 , 1 , 0 , 5
      1 , 1 , 2 , 3 , 0 , 1 , 3
      -5 , 6 , -9 , -8 , 0 , 0 , 0 ...
];
format = '%8.3g';
fprintf('%8s', 'x1', 'x2', 'x3', 'x4', 's1', 's2', 'b', '\n');
r = 1; c = 2;
B = pivot(A, r, c, format);
```

**Problem:** Define  $X = \mathbf{R} \times \mathbf{R}_+$  and  $Y = \mathbf{R}_+ \times \mathbf{R}_+ \times \mathbf{R}$ . We are given the following Lagrangian  $L : X \times Y \rightarrow \mathbf{R}$ :

$$L(x_1, x_2, y_1, y_2, y_3) = 3x_1 + 2x_2 + y_1(4 - x_1) + y_2(-8 - 5x_1 - 3x_2) + y_3(23 - 4x_1 - 2x_2)$$

We define the extended primal objective  $f : X \rightarrow \bar{\mathbf{R}}$  and dual objective  $g : Y \rightarrow \bar{\mathbf{R}}$  by

$$f(x) = \inf\{L(x, y) \mid \text{w.r.t } y \in Y\} \quad , \quad g(y) = \sup\{L(x, y) \mid \text{w.r.t } x \in X\}$$

(1) Write down a description of the set (call it  $U$ ) where  $f(x)$  is equal to  $-\infty$ . (2) Write down the value of  $f(x)$  for  $x \in X$  and  $x \notin U$  (in terms of  $x_1$  and  $x_2$  and with any inf). (3) Write down a description of the set (call it  $V$ ) where  $g(y)$  is equal to  $+\infty$ . (4) Write down the value of  $g(y)$  for  $y \in Y$  and  $y \notin V$  (in terms of  $y_1, y_2$  and  $y_3$  and with any sup).

**Solution:**

1. The variable  $y_1 \in \mathbf{R}_+$ , hence the inf with respect to  $y_1$  will be  $-\infty$  if and only if  $4 - x_1 < 0$ . The variable  $y_2 \in \mathbf{R}_+$ , hence the inf with respect to  $y_2$  will be  $-\infty$  if and only if  $-8 - 5x_1 - 3x_2 < 0$ . The variable  $y_3 \in \mathbf{R}$ , hence the inf with respect to  $y_3$  will be  $-\infty$  if and only if  $23 - 4x_1 - 2x_2 \neq 0$ . Thus the set  $U$  is given by

$$U = \{x \in X \mid 4 - x_1 < 0, \text{ or } -8 - 5x_1 - 3x_2 < 0, \text{ or } 23 - 4x_1 - 2x_2 \neq 0\}$$

2. If  $x \in X$  and  $x \notin U$ , then  $f(x) = 3x_1 + 2x_2$ .
3. Refactoring the terms in the Lagrangian we have

$$L(x_1, x_2, y_1, y_2, y_3) = 4y_1 - 8y_2 + 23y_3 + x_1(3 - y_1 - 5y_2 - 4y_3) + x_2(2 - 3y_2 - 2y_3)$$

The variable  $x_1 \in \mathbf{R}$ , hence the sup with respect to  $x_1$  is  $+\infty$  if and only if  $3 - y_1 - 5y_2 - 4y_3 \neq 0$ . The variable  $x_2 \in \mathbf{R}_+$ , hence the sup with respect to  $x_2$  is  $+\infty$  if and only if  $2 - 3y_2 - 2y_3 > 0$ . Thus the set  $V$  is given by

$$V = \{y \in Y \mid 3 - y_1 - 5y_2 - 4y_3 \neq 0, \text{ or } 2 - 3y_2 - 2y_3 > 0\}$$

4. If  $y \in Y$  and  $y \notin V$ , then  $g(y) = 4y_1 - 8y_2 + 23y_3$ .