MATH 407 QUIZ	August 7, 2008	Name (Please print):
Problem 1	40 points	______________________________

Problem: We are given the optimization problem

$$\begin{array}{lll} \text{maximize} & x_1 + x_2 & \text{w.r.t} \quad x \in \mathbf{R}^2_+ \\ \text{subject to} & x_1 + 2x_2 & \leq 2 \end{array}$$

1. Write down the two linear equations corresponding to each vertex of the feasible region corresponding to the problem above.
2. Write down the objective value corresponding to each of the vertices and which of the vertices correspond to the optimal value for the objective function.

Solution:

1. The only possible linear equations for this problem are

$$x_1 = 0 \ , \ x_2 = 0 \ , \ \text{and } x_1 + 2x_2 = 2$$

Each of the possible pairs of these equations, that has a unique feasible solution, is

$$\begin{array}{rrcr} \text{First vertex} & x_1 = 0 & , & x_2 = 0 \\ \text{Second vertex} & x_2 = 0 & , & x_1 + 2x_2 = 2 \\ \text{Third vertex} & x_1 + 2x_2 = 2 & , & x_1 = 0 \end{array}$$

2. The vertex points and corresponding value of the objective $x_1 + x_2$ are

	Vertex	**Objective**
First Vertex	(0, 0)	0
Second Vertex	(2, 0)	2
Third Vertex	(0, 1)	1

Since the optimial value for the objective must occur at a vertex, the point (2, 0) is optimal and it is the only vertex that corresponds to the optimal value for the objective.

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Problem 2	40 points	______________________

Problem: We are given the optimization problem

$$\begin{array}{rll} \text{maximize} & x_1 + x_2/2 + x_3 & \text{w.r.t} \quad x \in \mathbf{R}^3_+ \\ \text{subject to} & 2x_1 + x_2 + 2x_3 & \leq 2 \end{array}$$

1. Write down the three linear equations corresponding to each vertex of the feasible region corresponding to the problem above.
2. Write down the objective value corresponding to each of the vertices and which of the vertices correspond to the optimal value for the objective function.

Solution:

1. The only possible linear equations for this problem are

$$x_1 = 0\ ,\ x_2 = 0\ ,\ x_3 = 0\ ,\ \text{and } 2x_1 + x_2 + 2x_3 = 2$$

   Each of the possible triples of these equations, that has a unique feasible solution, is

$$\begin{array}{rrcrcr} \text{First vertex} & x_1 = 0 & , & x_2 = 0 & , & x_3 = 0 \\ \text{Second vertex} & x_2 = 0 & , & x_3 = 0 & , & 2x_1 + x_2 + 2x_3 = 2 \\ \text{Third vertex} & x_3 = 0 & , & 2x_1 + x_2 + 2x_3 = 2 & , & x_1 = 0 \\ \text{Fourth vertex} & 2x_1 + x_2 + 2x_3 = 2 & , & x_1 = 0 & , & x_2 = 0 \end{array}$$

2. The vertex points and corresponding value of the objective $x_1 + x_2/2 + x_3$ are

	Vertex	**Objective**
First Vertex	(0, 0, 0)	0
Second Vertex	(1, 0, 0)	1
Third Vertex	(0, 2, 0)	1
Fourth Vertex	(0, 0, 1)	1

   Since the optimial value for the objective must occur at a vertex, the vertices (1, 0, 0), (0, 2, 0), and (0, 0, 1) are optimal and these are all the vertices that correspond to the optimal value for the objective.